PARALLEL ALGORITHMS USING PVM FOR THE EVOLUTION OF INTERFACES

M. Falcone [†], P. Lanucara [‡], C. Truini [†]

[†] *Dipartimento di Matematica, Univ. di Roma "La Sapienza", ROMA*
[‡] *CASPUR, c/o Univ. di Roma "La Sapienza", ROMA*

ABSTRACT

We deal with some parallel algorithms for the numerical approximation of the front propagation problem. The model describe an interface moving in the normal direction with velocity $c(x) > 0$.

I. THEORY

Our model is based on the so called "level set" formulation (see e.g. [B], [OS], [ES1], [ES2]) and requires the numerical solution of the first-order Hamilton–Jacobi equation

$$\begin{cases} \varphi_t + c(x) \mid \nabla\varphi \mid = 0 \\ \varphi(x,0) = \Phi(x) \end{cases}$$

where Φ is a continuous function changing sign on the boundary of the initial configuration Ω.

The need for parallel algorithms is essentially due to the fact that an accurate computation of the interfaces requires a huge number of nodes, particularly for 3-D problems, since the equation is non linear and the geometry of the interfaces can be very irregular (typically we look for generalized solutions in the viscosity sense [Li]).

The above problem can be considered as a test for the parallel solution of more general Hamilton–Jacobi equations which are connected to a wide range of models and applications in control theory, image processing and mathematical physics (see [Li] and references therein). Particularly in control theory problems coming from real applications are set in a space of high dimension ($\mathbb{R}^n$ where $n >> 3$).

We present some convergence results and numerical experiments related to the application of a domain decomposition strategy to the above equation. Our formulation ([FLS], [CFLS]) can easily take into account a general subdomain decomposition obtained by means of an unstructured grid generation in each subdomain.

Our approximation scheme

$$\begin{cases} w(x_i) = \min\limits_{a\in A}[\mathrm{e}^{-h} w(x_i - hc(x)a)] + 1 - \mathrm{e}^{-h} \qquad \text{for } x_i \in \mathbb{R}^n \setminus \Omega \\ w(x_i) = 0 \qquad \text{for } x_i \in \Omega \end{cases}$$

is based on dynamic programming and it can be interpreted as a generalized method of characteristics (see for details [F], [FGL], [FF]).

We start from a local formulation of the problem based on a variable time step technique which permits the splitting of the original problem in Q into a number of problems belonging to sub-sets of manageable size Q_r. This strategy is well suited for the implementation of the algorithm on parallel machines.

II. PARALLEL IMPLEMENTATION

At each iteration of the algorithm every set Q_r of the decomposition send the boundary data to his neighbouring subdomains. The global solution in Q is obtained in the limit.

We examine the problems connected to the implementation of the above algorithm on parallel computers and in particular on MIMD machines with distributed memory using PVM (the numerical experiments have actually run on the cluster of 8 DEC/Alpha stations at CASPUR).

Two different kinds of domain decomposition have been adopted for the front propagation problem in $\mathbb{R}^3$.

The first strategy consists in a slice subdivision of the domain. The advantage of this decomposition is that the communication is required only between neighbouring subdomains (just two in this case). As a consequence, the communication part of the algorithm is simple but there is a risk of a break down of performances due to an unbalanced computation load.

The second strategy consists in a more tricky subdivision of the domain, essentially a chess-board subdivision. The exchange of informations between the processor is more complicated but the amount of data transferred in the communication phase is lower than in the first strategy and it results in a more balanced computation load.

We present an analysis of the numerical results also comparing the performances of the two strategies.

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Authors Address:

Dipartimento di Matematica, Università di Roma "La Sapienza", P. Aldo Moro 2, 00185 Roma, *e-mail: falcone@sci.uniroma1.it* and *truini@xantia.caspur.it.*

CASPUR, Consorzio per le Applicazioni del Supercalcolo per Università e Ricerca, Università di Roma "La Sapienza", P. Aldo Moro 2, 00185 Roma, *e-mail: lanucara@sci.uniroma1.it.*